Convergence analysis of the fast iterative shrinkage-thresholding algorithm

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1 Background

Many practical applications require to minimizing a convex function $F\colon \mathbb{R}^n \to \mathbb{R}$. Probably the most basic method for minimizing F is the gradient descent method, where an iterative sequence $(x_n)_{n\in\mathbb{N}}$ is constructed by

$$x_{n+1} = x_n - s_n \nabla F(x_n) \quad \text{for } n \in \mathbb{N}\,.$$

Here ∇F is the gradient of F and s_n is a step size. Some drawbacks of the gradient descent method are the following:

- ⇨ The sequence $(x_n)_{n\in\mathbb{N}}$ converges only slowly;
- ⇨ The algorithm requires F to be smooth.

In this this thesis we study the fast iterative shrinkage-thresholding algorithm (FISTA) introduced in [1] that overcomes both drawbacks of the gradient descent method while almost keeping its simplicity.

2 Aims of the bachelor thesis

In [1] the authors introduce the FISTA for minimizing a function that an be written in the form $F(x) = f(x) + g(x)$ where f and g are convex and p possibly non-smooth. They showed the improved convergence rate $\mathcal{O}(1/k^2)$ in the function values.

In this bachelor thesis, following [1] the FISTA algorithm is introduced and the convergence rate $\mathcal{O}(1/k^2)$ derived. The FISTA is based on an accelerated version of the gradient method introduced by Nesterov in [2]. Connections of the FISTA to [2] will therefore also established.

References

[1] A. Beck and M. Teboulle. A fast iterative shrinkage-thresholding algorithm for linear inverse problems. *SIAM J. Imaging Sci.*, 2(1):183–202, 2009.

[2] Y. Nesterov. A method of solving a convex programming problem with convergence rate $O(1/k^2)$. *Soviet Math. Dokl.*, 27(2):372–376, 1983.