Numerical inversion of a broken ray transform

Project for Bachelor Thesis
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1 Background

Inversion of Radon transforms is the mathematical basis of computerized tomography (CT). For example, the classical Radon transform is the basic tool for standard CT. More recent tomographic modalities are often based on different Radon transforms. In this bachelor thesis we consider a broken-ray Radon transform, which is relevant for single scattering optical tomography.

2 Aims of the bachelor thesis

As illustrated in Figure 2.1, the broken ray Radon transform $\mathbf{R}f$ we consider integrates a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ supported in the unit disc $D = \{x \in \mathbb{R}^2 \mid \|x\| = 1\}$ over all broken rays $\mathrm{BR}(u,t)$ with $u \in \partial D$ and $t \in (-1,1)$.

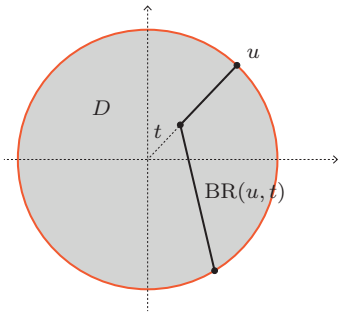

Figure 2.1: The broken ray $\mathrm{BR}(u,t)$ starts at the point $u \in \partial D$, points towards the origin and makes a turn at distance t from the origin in some fixed direction. The broken ray Radon transform $\mathbf{R}$ maps a function to its integrals over all these broken rays.

In [1] an efficient numerical method for inverting the broken ray transform (reconstructing f form $\mathbf{R}f$) has been derived. In this bachelor thesis, basic properties of the above broken ray transform are studied, and the inversion method of [1] is presented and numerically implemented in MATLAB.

References

[1] G. Ambartsoumian and S. Roy. Numerical inversion of a class of broken ray transforms arising in single scattering optical tomography. Preprint, arXiv:1510.03557, 2015.