Product integration for Abel integral equations

Project for Bachelor Thesis
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1 Background

Many practical applications, for example various tomographic techniques, yield to the solution of an Abel type integral equation,

$$\int_0^t \frac{k(t,s)}{(t-s)^\mu} f(s)\,\mathrm{ds} = g(t) \qquad \text{for } t \in [0,1]\,. \tag{1.1}$$

Here $\mu \in (0,1)$ is some parameter, $k(t,s)$ a given smooth kernel, and $g(t)$ are the given data. The goal is to recover the unknown function f.

In the special case that $k(t,t) = 1$ for all $t \in [0,1]$, the integral equation (1.1) can be solved explicitly. In the general case however, such an explicit solution is not available and numerical method have to be applied to approximate the solution of (1.1).

2 Aims of the bachelor thesis

In [1, Chapter 10] some methods for numerically solving (1.1) are presented. In this bachelor thesis some of these methods will be described. In particular, the product integration methods based on the trapezoidal and the mid point rule will be described and implemented in Matlab.

Further, some aspects of the convergence analysis of [2, 3] will be summarized and critically analyzed.

References

[1] P. Linz. *Analytical and numerical methods for Volterra equations*, volume 7 of *SIAM Studies in Applied Mathematics*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1985.

[2] R. Weiss. Product integration for the generalized Abel equation. *Math. Comp.*, 26:177–190, 1972.

[3] R. Weiss and R. S. Anderssen. A product integration method for a class of singular first kind Volterra equations. *Numer. Math.*, 18:442–456, 1971/72.